

# Re: looking for a predicate hierarchy

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- *From:* "Dmitry A. Kazakov" <[mailbox@xxxxxxxxxxxxxxxxxxxx](mailto:mailbox@xxxxxxxxxxxxxxxxxxxx)>
  - *Date:* Tue, 2 Jan 2007 14:14:19 +0100
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On Mon, 1 Jan 2007 18:35:22 +0100 (CET), V.J. Kumar wrote:

"Dmitry A. Kazakov" <[mailbox@xxxxxxxxxxxxxxxxxxxx](mailto:mailbox@xxxxxxxxxxxxxxxxxxxx)> wrote in [news:85gat69gqvtq\\$.11ps8kt0xar5e\\$.dlg@xxxxxxxxxxxx](mailto:news:85gat69gqvtq$.11ps8kt0xar5e$.dlg@xxxxxxxxxxxx):

These are only bounds, they need not to be exact. You can always say that if  $x$  is not contradictory then it is in  $[0,1]$ . Do you need to substantiate 0 and 1?

You seem to have misunderstood. What I am saying is that the interval is defined by two crisp real numbers that correspond to two membership functions  $F_-$  for the lower boundary and  $F_+$  for the upper boundary according to Atanassov. I do not see why using two crisp numbers like the judgement 'ball\_is\_red =  $[0.7..0.75]$ ' is any better than using one 'ball\_is\_red=0.7'.

No. It is

ball is red = 0.7 vs.  
ball is red \*in\*  $[0.7..0.75]$

The point is that equality = is replaced by inclusion, which moves uncertainty from the value to the relation between "ball is red" and a value of.

Instead of using one crisp number we use two so this way of doing thing looks even more suspect than before.

It is only syntactically two numbers. Semantically it is an uncountable set of numbers.

When you say that  $x$  is [intuitively] more red and small than  $y$ . That assumes some sort of additive measure of  $\text{Red} \setminus \text{Small}$ .

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It does not assume anything of the kind. I think you've got it backwards. We want to model some fragment of the 'real world' so we look for some tools, not the other way around, we take a tool and try to make reality fit the tool, be it fuzzy logic, possibility theory, or whatever.

First of all, arguments to "real world" are automatically disqualified as philosophical. (:-))

With the tools at our disposal, one would expect that x being red and small is a better choice than y, if the tools say 'no', than they are not good tools.

I just tried to model your notion of "better" choice. Additivity was a formalization of having  $s(x \text{ in Red} \wedge \text{Small}) > s(y \text{ in Red} \wedge \text{Small})$ . Additivity depends on the elementary events/outcomes which compose x, y, Red, Small. Only when these elements are mutually independent we might get something like an additive measure. The opposite situation is when they are nested. In that case the measure would be max/min. In "real world" you might have nether.

Not every measure is additive. Also, if you think that additive measure is universally intuitive, then consider obvious:

In fact, with the above example, I used a non-additive measure. According to Zadeh, the membership function \*is\* a possibility distribution which is a pretty reasonable point of view as membership degrees are clearly non-additive. Just to make sure we are on the same page, in simple terms measure additivity is just the property that  $m(A \text{ OR } B) = m(A) + m(B)$  where A and B are some sets (see Kolmogoroff axioms).

Yes, that is what I meant.

Clearly, the property has not been (and could not be) used in the red balls example.

We need a clear model behind the example. Further, possibility can always be used because  $\text{pos}(x \text{ in Red} \wedge \text{Small}) \leq \min\{\text{pos}(x \text{ in Red}), \text{pos}(x \text{ in Small})\}$  is universally correct. From that you can get that both

$$\begin{aligned}\text{pos}(x \text{ in Red} \wedge \text{Small}) &\leq 0.5 \\ \text{pos}(y \text{ in Red} \wedge \text{Small}) &\leq 0.5\end{aligned}$$

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So? You are still free to hold  $x$  in  $\text{Red} \wedge \text{Small}$  for more possible, if you could add your intuition as an evidence. That formally means, that you possessed some additional knowledge about relations between  $x$ ,  $y$ ,  $\text{Red}$  and  $\text{Small}$ . This might improve the estimations above.

Further, any upper bounds don't tell anything about the actual relation between  $x$  and  $y$  being in  $\text{Red} \wedge \text{Small}$ , only about the possibility of. You need lower bounds, the necessities, to make any conclusions. But even then, you would have only two estimations of  $x > y \mid \text{Red} \wedge \text{Small}$ , i.e. the upper and lower bounds for the possibility and necessity that the measure of  $x$  is greater than the measure of  $y$  on the set  $\text{Red} \wedge \text{Small}$ .

[This is exactly the same situation as with probabilities. Once you get in, you can't obtain nothing but measures.]

$$p(x \text{ is Red}) \wedge p(x \text{ is Red}) > p(x \text{ is Red})$$

Is it? Does repeating a wrong statement make it more valid? That would be rather a propaganda, than logic. (:-))

I do not see the point. According to the red balls example,  $p(x \text{ is Red}) / \setminus p(x \text{ is Red}) = p(x \text{ is Red}) = 0.5$

Yes, because that is the property of  $p = \max$ . If you took anything else, like empirical  $t$ - and  $s$ -norms, you would get paradoxes like above. The only reasonable measure which allows to universally decompose  $s(A \wedge B)$  into  $f(s(A), s(B))$  independently on how  $A$  and  $B$  are related to each other is possibility (necessity). All others would require independence analysis of  $A$  and  $B$ . The price for that is that pos/nec give only estimations, so, in general you cannot judge about  $x > y$  in your example. Therefore, your argument is illegal. The example just does not tell anything certain about  $x > y$  or  $x = y$ .

Take another example, you need to hire someone as a lead Java developer who knows both Java, naturally, and SQL. You have two persons with (1, 0.5) and (0.5, 0.5) qualifications. Who are you going to hire? The fuzzy logic says it does not matter (see the ball example). What measure/possibility distribution/membership function would you come up with to solve the problem correctly?

OK, Let you were interested in a person who knows both Java and SQL. That formally means that you want to know the measure of  $\text{Java} \wedge \text{SQL}$  provided given person  $x$ , i.e.

$$\text{pos}(\text{Java} \wedge \text{SQL} \mid x)$$

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nec (Java^SQL|x)

The latter tells us how it is possible that the person x would not know Java and SQL:

nec (Java^SQL|x) = 1 - pos (~(Java^SQL)|x)

This model considers anything outside Java^SQL as non-asset. Is that really true? If yes, then the answer was correct. Because no matter how much were Java^~SQL|x, that would not be an asset.

But consider this:

pos (Can\_learn\_SQL\_in\_a\_week ^ Java ^ ~SQL | x)

Does this matter to the choice? If yes, then Java^SQL was a wrong objective. Inadequate models provide inadequate answers.

Hmm, without going into philosophical issues about merits of contradictory inference (not to be mixed with inference from contradiction), but purely technically, less inference paths you take, smaller is the set of consequences. So inference under certainty cannot explode more than one under certainty + contradiction.

Could you explain ? I am not sure what you mean by that. An example would be nice.

Consider the implication you referred.

a> | T 0 1 \_|\_
-----+-----
T | T 0 1 \_|\_
0 | 1 1 1 1
1 | T 0 1 \_|\_
\_|\_ | 1 1 1 1

When reasoning under certain truth, the only valid paths were 1=(x a> y):

a> | T 0 1 \_|\_
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T | - - 1 -
0 | 1 1 1 1
1 | - - 1 -
_ | _ | 1 1 1 1

```

For reasoning under contradiction or else certain truth [  $1=(x \text{ a} > y)$  or  $T=(x \text{ a} > y)$ : ] it would be:

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a > | T 0 1 _ | _
-----+-----
T | T - 1 -
0 | 1 1 1 1
1 | T - 1 -
_ | _ | 1 1 1 1

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Here we can deduce more:  $\{T, T\} \models (T=(T \text{ a} > T))$  and  $\{1, T\} \models (T=(1 \text{ a} > T))$ . But this cannot influence explosiveness of inference under  $1=(x \text{ a} > y)$ , in the sense, that if  $1=(x \text{ a} > y)$  were explosive then  $1=(x \text{ a} > y)$  or  $T=(x \text{ a} > y)$  would be as well.

No, explosiveness is defined as \*all\* the formulas hold as a consequence of a certain formula/set of formulas. You've just demonstrated a valid derivation, not explosivity.

The point is why \*more\* valid derivations should make anything else non-explosive?

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 Regards,  
 Dmitry A. Kazakov  
<http://www.dmitry-kazakov.de>