Re: VOTE on whether 1/oo = 0

Source: http://coding.derkeiler.com/Archive/General/comp.theory/2004−07/0355.html

---

From: David W. Cantrell (*DWCantrell_at_sigmaxi.org*)
Date: 07/14/04

```
Date: 14 Jul 2004 17:53:47 GMT
```

"|−|erc" <gotch@beauty.com> wrote:
> > *Does 1/oo = 0 ?*
>
> *Thanks to all participants and Kent for his insightful rebuttal on voting*
> *and maths.*

For anyone in sci.logic who may be confused, Herc began his poll in sci.math two days ago, with the following post:

−−−−−−−−−−−−−−−−−−−−−−−−−−−−−
"|−|erc" <gotch@beauty.com>
wrote:
> *Please don't justify your answer or cite reasoning to detract from the*
> *next voters opinion.*
>
> *Does 1/oo = 0 ?*
>
> *oo = infinity*
> *0 = zero*
> *= = equals*
> *1 = one*
>
> *If you're a regular professor here let others vote 1st so as not to*
> *influence the result.*
>
> *Just post YES or NO to be counted*
>
> *Thanks*
> *Herc*
−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−−

I'll presume that he thinks it's OK now for "a regular professor here" to reply. And I feel somewhat compelled to do so since I see that my own words have been "taken in vain", so to speak.

> *No is what I thought that is why I was surprised when Barb Knox claimed*
> *yes.*

\>
> With all the other bizzare interpretations of maths I was taught that
> people around here take as correct I had to check.

You thought a poll excluding any "regular professor here" would be a good way to check! Bah. :−(
I will say, however, that the results of the poll do not really surprise me, and yet, as explained below, they sadden me somewhat.

Suppose you had taken a poll asking "Does i^2 = −1?", for example. I doubt that anyone would have responded "No". Why? Because they would naturally have interpreted the question in a reasonable context, _one in which the question made sense_! Of course, had someone wished to be difficult, they could have said "No" and justified their answer easily by saying "In the real number system, i does not exist and so it is meaningless to speak of i^2." But that's not a good answer and justification. Nobody in their right mind would naturally choose to limit themselves to the real number system when asked "Does i^2 = −1?" Thus, it saddens me that so many respondents to the poll chose to answer using only a context in which 1/oo is nonsensical, and then answered "No" accordingly. If asked "Does 1/oo = 0?", the natural thing to do −− at least for me −− is to choose to answer within a context in which all parts of the question _make sense_! In _any_ such context known to me, the answer is "Yes."

Examples of such contexts include the one−point extensions of the real and complex numbers. An even simpler example is the system [0, oo].

> > >> >Or does 1/oo = 0 now?
> > >>
> > >> It certainly does. For example, have a look at
> > >> <http://mathworld.wolfram.com/AffinelyExtendedRealNumbers.html>,
> > >> item (7). Didn't you already know that?

Yes, that's also an example. But the two−point extension of the reals is not quite as nice to work with here because 1/0 is undefined in that extension, and so we cannot say that 0 and oo are reciprocals of each other there.

> > >"these improper elements are not real numbers, and that this system of
> > >extended real numbers is not a field."
> >
> > Apparently your super−powers do not include reading comprehension.
> > From the same web page:
> > 'The above statements which define results of arithmetic operations on
> > oo may be considered as abbreviations of statements about determinate
> > limit forms. For example, −(+oo) = −oo may be considered as an
> > abbreviation for "If x increases without bound, then −x decreases
> > without bound."'

I was careful in wording that. I said "...may be considered as abbreviations..." I did not say "...are abbreviations..." or "...must be considered as abbreviations...", for example. The fact that "The above statements which define results of arithmetic operations on oo may be considered as abbreviations of statements about determinate limit forms." does not mean that it is in any way _best_ to do so. And indeed, at the end of the paragraph from which that quotation was taken, I mention that some authors choose to take 0*oo = 0. That _cannot_ be considered to be an abbreviation of a statement about a determinate limit form since the corresponding limit form is indeterminate.

> *Which means 1/oo = 0 is an abbreviation for lim(x−>oo) 1/x = 0*

As I had said, you _may_ think of it that way. But I prefer to think of lim(x−>oo) 1/x = 0 as being the reason why we clearly want to have 1/oo = 0 in an extended system such as [0, oo]. Within such a system, once its elements and operations have been defined, 1/oo = 0 _per se_. That equation need not be regarded as being merely an abbreviation.

The only time that 1/oo = 0 _must_ be regarded as being merely an abbreviation is when we are dealing with systems such as the (unextended) real or complex numbers. They lack an infinite element, and so 1/oo, taken literally, is nonsensical in them.

> *That first "=" is not equals.*

Within an extended number system, it does represent equality.

> *You are better off using notation 1/oo <=> 0*

I wouldn't say so. That notation would probably be confusing. Better to work within an extension where "=" means "equals".

> *I do have a liberal perspective on the topic also.*
>
> *IF you define division by infinity to start with*
> *THEN you could assign that result as 0.*

I advise against doing that.
It's better to define the elements of the extension and the appropriate operations _in general_ first. Done correctly, 1/oo = 0 would then follow as naturally as, say, 6/2 = 3. There would be no need to consider "division by infinity" as being a special case.

> *But like someone pointed out a/b = c <−> a/c = b which doesn't work*
> *with division by 0.*

Certainly 0/5 = 0 doesn't imply that 0/0 = 5, for example. But more pertinently for this thread, note that (1/b = c iff 1/c = b) _does_ work perfectly well in the one−point extensions of the real and complex numbers, and in the system [0, oo]. That's one reason why, in such systems, it's reasonable to call 0 and oo _reciprocals_ of each other, despite the fact that they are not multiplicative inverses.

Maybe you didn't correctly state what "someone pointed out". Perhaps they had instead said something like (a/b = c implies a = b*c). That implication is true in the real and complex number systems, but false in their extensions, in which division is not defined directly in terms of multiplicative inversion.

David W. Cantrell