Re: THIS STATEMENT HAS NO PROOF IN ANY SYSTEM = true or false?

Source: http://coding.derkeiler.com/Archive/General/comp.theory/2005−01/0452.html

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Date: 01/15/05

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Date: 15 Jan 2005 01:07:07 −0800
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Torkel Franzen wrote:
> *poopdeville@gmail.com writes:*
>
> > *Relevant to the existence of truth vis a vis intended models versus*
> > *plain−old models.*
>
> *That's a bit cryptic, and insufficient to explain such a startling*
> *statement. Since models of ZFC is a very specialized topic, which*
> *is irrelevant to the work of most mathematicians, whereas every*
> *mathematician is familiar with groups, saying that there is*
> *"no relevant distinction" between the two subjects calls for a bit*
> *of elaboration.*

Fair enough. I'll see if I can. This is a bit hazy to me as well.
:−)

There are two fairly obvious distinctions between the two, other than the fact that the axioms of ZF and of group theory are different in content. The first is the one you noted: as a matter of mathematical practice, mathematicians are more familiar with models of groups than with models of ZF. The second is more linguistic in nature. A mathematician asked if the axioms of group theory are true would likely note, as we have all noted, some awkwardness in the way the question was phrased. This is related to the first distinction in that even if the mathematician isn't thinking about interpretations and structures and all that jazz, he is thinking about the axioms being true *of something;* namely, particular groups. (Which of course are models in the logical sense)

Paraphrasing, amongst other things, Tim Chow asked if AC and the other axioms of ZF are true. Unless he was using a non−model−theoretic use of the term "true," the ZFC is just as true as the group axioms, since we can exhibit models for both sets of axioms. Via forcing, we can construct a model where ZF holds but AC fails, and similarly, we can construct a model where only two out of the three group axioms hold. Neither of the distinctions is relevant to the existence of such models.

If Tim wasn't using the model−theoretic notion of truth above, then he wasn't particularly clear what he meant. One reasonable disambiguation is whether or not the "intended model" of set theory satisfies the axioms −− whether or not ZF satisfies our intuitive notions of what it means to be a set. As evidenced by the plethora of intuitive notions of sets on sci.logic, there is no unique intended model. This relates to the point I made regarding the acceptance of AC among mathematicians −− although there are many intended models, there is enough overlap across intended models of set theory (either by natural intuition, education, indoctrination, or some other sociological phenomenon) so that AC is accepted by a majority of, but not all, mathematicians.

Another reasonable disambiguation is that Tim was referring to truth simpliciter −− a phrase that I've heard others use and describe, but that I find meaningless.

'cid 'ooh