Re: does sqrt(2) exist in CM?

Source: http://coding.derkeiler.com/Archive/General/comp.theory/2005−02/0532.html

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examachine_at_gmail.com
Date: 02/16/05

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Date: 16 Feb 2005 03:41:24 −0800
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tchow@lsa.umich.edu wrote:
> In article <1108500848.830038.313970@z14g2000cwz.googlegroups.com>,
> <examachine@gmail.com> wrote:
> >I read the proofs and studied the construction a long time ago.
Should
> >I be now "proving" that to you, because of Torkel's silly posts?
>
> I've studied lots of mathematics a long time ago that I later forgot
the
> details of. But when the time comes to discuss them, I have to go
back
> and review the material to make sure I don't spout nonsense.
>
> What does it matter whether Torkel is a gadfly or if you thought I
was
> being condescending? Getting the statement of Chaitin's theorem
right
> is a worthwhile thing, so why not do it?

There is no need for me to explain anything. Chaitin already explains the theorems perfectly well. The significance of his most refined incompleteness result, that you need n+c bits of algorithmic entropy to calculate n scattered bits of Omega (where c depends on universal computer chosen) is closely tied to the diophantine construction for Omega. This is best explained by Chaitin himself:

http://www.umcs.maine.edu/~chaitin/unknowable/ch6.html

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I use the work of M. Davis, H. Putnam, J. Robinson, Y. Matijasevic and J. Jones on Hilbert's 10th problem to encode the bits of O in a diophantine equation. My equation is 200 pages long and has 20,000 variables X1 to X20000 and a parameter K. The algebraic equation

L(K, X1, ..., X20000) = R(K, X1, ..., X20000)

has finitely or infinitely many natural number solutions (each solution is a 20,000−tuple with the values for X1, ..., X20000) if the Kth bit

of O is, respectively, a 0 or a 1. Therefore determining whether this
equation has finitely or infinitely many solutions is just as difficult
as determining bits of O.

There is another construction that's easier, but this will do.

As you can see this definition is not bombastic at all, perhaps people prefer dull mathematicians, I personally find his writing style well.

What does this mean to a mathematician not very familiar with theory of computation? Chaitin's view is that his incompleteness theorem (e.g. irreducibility of Omega) is directly exhibited in elementary number theory by this construction, in similar vein to the demonstration of Godel's incompleteness theorems as arithmetic facts.

This is what I mean by a "diophantine problem", it is a diophantine equation with a free variable "K". To "solve" this "diophantine problem" means plugging an arbitrary K and trying to solve it.

Assuming that the mathematician who wants to solve this problem for arbitrary K is a finite mechanism, IT TRIVIALLY FOLLOWS that Chaitin's incompleteness theorem means that there is an absolutely undecidable problem in elementary number theory. Furthermore, it exhibits randomness in elementary mathematics, because the solution not governed by a rule (although it is computable in the limit, here computable meaning the precise definition which Ramsay and others gave). Is this more drastic than Godel's and Turing's construction? In one broad sense, these all deal with the same problem: halting problem, but I find the theorems of Chaitin stronger than Godel's, because they quantify what you can do "at best" (e.g. they are upper bounds).

On the other hand, since I see you as an interested person, I would like to point out that Raatikainen's critique of one of the lower bound results of Chaitin (akin to Godel's theorem) is not justified, because it depends on a "philosophical" point of view that is downright ignorant of computation. The examples in that paper are fairly trivial cases that are covered by AIT, even if some logicists cannot understand that. The discussion on this thread was much more informed than the unwarranted talk about infinite-size universal computers, and all the nonsense in that paper that Raatikainen managed to publish.

Regards,

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