

## Re: Existence of mathematical entities (Re: Successor Axiom: on what grounds TF?)

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"Mattias Wikström" <mattias\_wikst71@hotmail.com> wrote in message news:1108779996.434915.244290@g14g2000cwa.googlegroups.com...  
Paul Holbach wrote:

Mattias,

You make an excellent criticism of PH, here. I have great respect for PH. Nevertheless, there is a clear problem surrounding the existential quantifier that is ignored by logicians.

If you look at Paul Halmos' papers on algebraic logic, you will find that existential quantification is closely associated with Kuratowski's closure axioms. If I turn to the ontological realm of formal set theory, the "utility" of the axiom of foundation/regularity for model construction cloaks the fact that the transitive closures that also arise from this axiom satisfy Kuratowski's closure axioms.

The history of "logicism"—as I understand it thus far—ignores "objectification."

One can verify this in both Dedekind and Frege. Moreover, when one turns to Russell or Carnap, you see the advent of what Shapiro calls "fictionalism."

Russell clearly refers to classes as "logical fictions." Carnap, building on

Whitehead and Russell, goes so far as to lament Cantor's definition of set on the basis that Cantor treats sets as objects. Rather, he takes classes to be understood as a "way of speaking."

Halmos' work is not some nonsense invoking categorial errors as modern advocates of first-order predicate logic seem to think. It arises as consequent to Tarski's work on cylindrical algebras, and, the Polish school has continued work in this area to the present day. In Czelakowski's book,

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"Protoalgebraic

Logic" you will find an interesting definition of a closure space.

Moreover,

one of the properties of closure spaces is that they are inductive.

For my part, I find it a bit strange that logicians are so concerned about consistency and then fail to see that the structures in which they are most interested are fundamentally paraconsistent—or, at least, seem to be.

When you make the observation,

"Whenever you see an object, you  
silently add it to the domain of existence.

you are expressing the basic presupposition underlying Cohen forcing in set theory. Cohen—or, more correctly, the modern characterization of Boolean-valued models and forcing languages—makes this explicit in a way that creates a nearly inconceivable ontology for anyone who takes set theory seriously.

Good catch.

- > *For irreferential singular terms the interpretation/reference function*
- > *clearly ought to remain undefined so as to avoid ontological hypertrophy:*
- >
- > *i("Pegasus") = ...*
- >
- > *"The problem, put crudely, is to make sure that things that don't exist don't end up existing after all. The predicate 'exists' fails to*
- > *apply to some things [\*]—what we have to ensure is that these things*
- > *don't turn out to exist in some attenuated or second-class way: they*
- > *simply don't exist."*
- >
- > *[McGinn, C. (2000). /Logical properties: Identity, existence, predication, necessity, truth/. Oxford: Oxford University Press. (p. 37)]*
- >
- > *[\*: I reject McGinn's particular formulation here because from "The predicate 'exists' fails to apply to some things" one might conclude that "There are some things to which the predicate 'exists' doesn't apply", which statement is considered definitely false by me.]*

Let  $o$  be an object. It seems to me (from what you have written elsewhere) that you want the following properties for the statement " $o$  exists":

A) " $o$  exists" should mean "there exists an  $x$  such that  $x=o$ ".

B) " $o$  exists" should be an analytic truth.

However, it seems to me that you cannot have A) and B) at the same

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time. I take "there exists an  $x$  such that  $x=o$ " to mean the same as " $o$  is within the domain of existence". Now, this statement can turn out true or false depending on what object  $o$  is and what domain "the domain of existence" is, and in this sense it is not analytic. The proof-theoretic rules that existential quantifiers obey simply are not enough to guarantee its truth.

I think what makes "there exists an  $x$  such that  $x=o$ " seem analytic is the way it is used by you and others. Whenever you see an object, you silently add it to the domain of existence. This not only makes it seem absurd to consider an object outside the domain of existence, it also serves to specify what the domain of existence is (as far as the rules for existential quantifiers are concerned, that domain can be any domain, and as far as I am concerned, it really can).

In addition to the properties A) and B) that I listed above, you sometimes seem to use:

C) " $t$  refers to  $o$ " entails " $o$  exists". (That is, reference is connected to existence: Whatever is referred to exists.)

Of course, C) is trivial given B), but again I think you must give up B) in order to have A).

A good way to see how my own ideas differ from yours is to consider a piece of text  $T$  that apparently describes an object. I do not hesitate to say that the text describes an object  $o$  (ontological hypertrophy is no problem for me, because I never say of objects I encounter that they exist), but for you there are two possibilities (assuming I have understood you):

i)  $T$  (or rather the relevant linguistic terms in  $T$ ) does not refer to any object.

ii)  $T$  does refer to an object  $o$ , and there exists an  $x$  such that  $x=o$ .

Mattias