

Re: Mathematical definition of automata?

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- *From:* markwh04@xxxxxxxxxx
 - *Date:* Fri, 08 Jun 2007 13:45:57 -0700
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On May 17, 5:55 pm, Vadim Tropashko <vadimtro_inva...@xxxxxxxxxx> wrote:

Is there automata model without start and stop state?

A state diagram: a labelled directed graph.

The path-closure of the graph has the following elements:

- (1) the set V of vertices of the original graph
- (2) an identity transition $I_v: v \rightarrow v$, for each vertex v in V
- (3) the labelled edges $p: v \rightarrow v'$ of the original graph
- (4) the product $(fg): v \rightarrow v''$ of any two edges $f: v \rightarrow v'$ and $g: v' \rightarrow v''$

of the graph.

- (5) (typed) identity property: $I_v f = f = f I_{v'}$, if $f: v \rightarrow v'$
- (6) (typed) associativity property: $(fg)h = f(gh)$, if $f: v \rightarrow v'$, $g: v' \rightarrow v''$, $h: v'' \rightarrow v'''$.

That defines none other than the mathematical structure known as a Category. A Category is a typed algebra modelled on the monoid: $1x = x = x1$; $(xy)z = x(yz)$, with the above-mentioned type restrictions. Categories are also used in developing the foundations of mathematics (so, some people go all glossy eyed over them at their mere mention as if there were anything more than a mere typed algebra), but that's neither important or relevant here.

The path closure construction is the standard method by which a Category is freely generated from a labelled directed graph. This is the typed analogue of constructing a free monoid X^* from the set of sequences of elements from X .

Nothing in the foregoing requires V to be a finite set. Each of the models higher up in the hierarchy beyond the FA defines a family of infinite state automata.

These ideas are developed fully in the following:

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The Untold Story of Formal Languages
Part 3: The Algebraic Representation of Automata
<http://federation.g3z.com/CompSci/index.htm#Untold3>

This section features the introduction of an algebra for state diagrams. Treating a state diagram as a graphical representation of a matrix, one can define the operations of addition and multiplication over them. The result is an extension of the cycle notation used for representing groups.

The classes of automata seen in classical formal language and automata theory may all be seen as instances of a general "infinite state automata" model. What distinguishes each class is (a) the factoring of its